Arbitrage possibilities in Russian spot and future markets
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Introduction

A stock market of any country is a very difficult object for assumptions and studies. The main reason is that prices in the stock market, in comparison to the market of material goods, are mainly defined by expectations of the participants in the trading process than by external objective economical factors. The internal factor is especially strong in the formation of developing markets, where it is most obvious that the stock and financial markets are relatively independent.

In the classical works it is assumed that there is a firm relationship between the stock prices and the expected discounted flow of dividends. This approach is not acceptable for the estimation of the Russian stock market, since the companies prefer to pay no dividends or the dividends are miserable. Thus in Russia there is no stable correlation between the price of a stock share and financial perspectives of its emitter. The speculative interests of the market participants and any information of possible merges of companies play the crucial role.

The Russian futures stock market has been actively functioning almost during two years till May 1998, when the main trading place --- the Russian stock market --- ceased to operate. This time is characterized by a rapid, almost five-times, increase in prices on the stock market and by an even more rapid decrease in them. How did the futures market react on this dynamics? To what extent could the futures price of contracts forecast the future spot price of stocks? The present paper tries to give answers to these questions.

In Section 1, to analyze the situation in the stock market in the period from September 1995 to September 1998, we divide the entire observation time into intervals each of which is characterized by it own political and economical situation in the country.

Due to the existing tradition, all the prices are in American dollars but the trade is carried out in Russian roubles. The exchange course is defined by the Central Bank of Russia. There were few main considerations that we used in the description of the dynamics of the Russian stock market. If for the most part the stock trade is carried out by Russian residents without any significant participation of foreign capital, then one hardly can expect any significant diversion in the dynamics of stock prices and that of the gold and currency reserves of the Central Bank. In the Russian market there is a high correlation between the RTS Index and the gold and currency reserves of the Central Bank. If, in addition, there is no direct correlation between changes in the reserves and the net foreign trade deficit, one can assume that the most important role in the Russian stock market is played by intentions of foreign investors and possible changes in the political situation are more important for them than for the national investors.

In this period, the short-term market of state bonds (GKO) and related debt instruments was a very important reference point in the financial market. To describe this very complicated market, we study the following three indices: GKO-90, GKO-180,

GKO-270, which are prices of some synthetic bonds with definite payment periods. Thus the time series describing the dynamics of synthetic bond indices, the RTS index, and the gold and currency reserves of the Central Bank form the basis for the descriptive analysis of the stock market

Looking for analogs of the Russian stock market among the stock markets of other countries, we paid our attention to the stock market in Venezuela whose dynamics is very similar to that in Russia.

In Section 2 of the present paper we study some theoretical aspects related to realization of simple arbitrage strategies. As far as we know, many of these strategies were successfully used by some broker companies in the Russian futures and spot markets. Here we also formulate some criteria, which allow one to decide, taking into account the GKO market and the borrowed capital market, whether a situation arising in the stock markets is favorable for using a particular arbitrage strategy.

In Section 3 we statistically analyze possible arbitrage strategies in the Russian stock market during the period from November 1996 to May 1998 by using, as an example, the two most liquid stocks of NK Lukoil and RAO EES. This analysis shows that there were rather long time periods during which pure arbitrage operations were possible, i.e., it was possible to «make money from air». The existence of this possibility made the Russian financial market very attractive, and one could hardly expect that banks would invest in the real sector of economy where the rate of return was not very large.

Moreover, there were rather long time periods when quasi-arbitrage operations were possible. The rate of return obtained in these operations by participants possessing a primary capital was significantly larger than that obtained in the GKO-market under the same costs.

Despite these facts, the futures market could provide average estimates of relative changes in the spot prices of stocks without any statistically significant deviations.

Thus the existence of a large number of possibilities for realization of arbitrage strategies does not contradict the fact that there are unbiased estimates of the rate of return via known prices of futures contracts.

1. History of the stock and internal State debt markets in Russia in 1995-98 .

The actual market history of the Russian economy originates from 1992--1994. Several fundamental laws in privatization and credit-money policy adopted in this period laid the foundation of the existing Russian market institutes.

It should be noted that, in practice, although a law is adopted, this does not mean that all existing financial and economical institutes and economy objects transform immediately. The process of constructing new market institutes is rather long. Only starting from 1995, one can say that the Russian market institutes and the market infrastructure operate in the same way as it is known in developed western countries.

Since each separate market simultaneously deals with numerous purchase-and-sell objects (stocks, bonds, futures contracts, etc.), one can completely describe the stock and GKO markets only by introducing some aggregate instruments, i.e., stock market indices.

There are many methods for constructing stock market indices, which differ in the base access (in quantity and the name of stocks included into the calculational set of the index) and in the calculational formulas. In many countries, several tens of different indices are sometimes calculated in the stock markets. These indices show, for instance, the dynamics of company stocks in different industry branches or of stocks comparable in liquidity (the daily trade volume). How to analyze the behavior of different stock market indices is the theme of a separate paper. Here we restrict ourselves to the study of the RTS index of the stock market and to three indices of the GKO--OFZ market.

The RTS index. The RTS index is being calculated from September 1, 1995. The RTS listing includes stocks of practically all the largest Russian companies (except for RAO "Gasprom"). The RTS index is weighted with respect to the company capitals and thus shows their market value. The index access referred to September 1, 1998 includes 88 stocks. The RTS index can be calculated by the recurrent formula

$$I_t = I_{t-1} * \frac{Cap_t}{Cap_{t-1}}$$

where Cap is the capitalization of the stock market at a corresponding time instant. At the initial time instant (September 1, 1995) the index was equal to 100.

During the four years under study, the RTS index is de-facto the most important factor of the entire Russian stock market for the majority of participants in the market. It is related to the large trade volume in the RTS (much more than the corresponding factor of the stock markets) and to a large amount of bonds listed in the RTS (this factor of the RTS is also larger than that of the stock markets).

In contrast to any unweighted (speculative) index, the RTS index is much more stable under sharp oscillations of prices of single stocks, since the main contribution to the index is made by companies with large capitalization. As a rule, the stocks of such companies are extremely tolerant to a momentary behavior of individual players.

To demonstrate an essential difference in the dynamics of the Russian and American stock markets, in Fig. 1 in Appendix we show the RTS index and the Dow-Jones average during the time period in question. The solid line shows the polynomial trends of these indices. A rapid increase and a more rapid decrease in prices in the stock market are special features of the state of affairs in the Russian stock market from September 1995 to October 1998.

The GKO-OFZ market indices. In contrast to the stock market, where the stock market indices are widely used, aggregate indices are less common in the state bond market. First of all, this is related to the fact that it is impossible to formulate a fixed access for such an index, since all GKO issues have a bounded lifetime. At the same time, the effect of studying several issues separately is very small, since they, first,

strongly depend on the behavior of the separate operators and, second, have a clear dynamics during their life, i.e., the price is increasing while the interest is decreasing.

Thus the main distinction between the «usual» stock market indices and the GKO indices is that the latter must have a «floating» access. In other words, the set of issues contained in the base access must vary constantly. For each separate issue, its payment period (in days) is a criterion for including it into the access (or for excluding from it). Since the most important factors characterizing the bonds (the price and the interest) depend on the payment period, it is not a lucky idea to include all bonds into the same index. One of possible solutions of this problem is to use several GKO-indices (for example, three), which characterize the short-term, medium-term, and long-term bonds, respectively. This distinction also has a historical background. During 1993—1996 there was a strict division of issues (three month bonds 21XXX, half-year bonds 22XXX, year bonds 23XXX). Thus we propose to use indices that, at each time instant, include bonds shown in Table1.

Table 1.

Index	Number of days till the repayment of bonds included into the index	Base period of maturity in days (T)
short-term index GKO_90	60-120	90
medium-term index GKO_180	150-210	180
long-term index GKO_270	240-300	270

To calculate the corresponding price index and the daily trade volume of all bonds included into the index, we calculate the price of a synthetic bond by the formula

$$P_T(t) = \frac{100 * 365}{Yield_T(t) * T}$$

$$Yield_T(t) = \frac{1}{\sum_{i=1}^{n} v_i(t)} * \sum_{i=1}^{n} Yield_i(t) * v_i(t)$$

Here T= 90, 180, or 270 for GKO_90, GKO_180, and GKO_270, respectively;

n is the number of GKO issues traded on the corresponding day t and having the date of maturity in the required interval (T-30; T+30),

P(t) is the price of the corresponding synthetic bond on the day t,

Vi(t) is the trade volume of the i-th issue on the day t,

365 is the number of days in the year,

100 is the (nominal) price of the bond.

The total volume of all traded bonds in a given interval is the trade volume for the desired synthetic bond.

Figure 1 shows how the four aggregate indices of the Russian stock market depend on time (compared during the same time period). The following time series are considered: the RTS index (the RTS variable), the gold and currency reserves of the Central Bank of the Russian Federation (the GCR_4 variable) in mill. Doll. USA, and the prices of GKPR_180 and GKPR_90 of the synthetic GKO with 180- and 90-day periods of payment. The scale of the variables GKPR_180 and GKPR_90 corresponds to the nominal price 100. We used the data observed from February 1, 1996, to August 14, 1998, i.e., to the time instant at which the Government defaulted on payments for short-term bonds. Since GKO-bonds were sold four days a week, all our data correspond to these days. For GCR-4 we approximate the Central Bank data linearly and then average over four days.

The dynamics of the Russian stock and bond market is subject to many factors both political and caused by the state of affairs in other sectors of the financial market, i.e., in the currency and state bond markets. It is worth noting that there is a high correlation between the stock market and gold and currency reserves of the Central Bank of the Russian Federation, as well as the stock markets in other countries. However, all these correlations are not stationary. Therefore, starting from political and economical factors that has been acting in Russia since 1995, we divide the observation time into stages. The passage from one stage to another is determined by certain events that happen outside the market but significantly affect the dynamics of its factors.

The natural bounds of these time intervals are important political and economical events that occurred in this period and significantly affect the participants in the Russian stock market. We divide the time-period from January 1995 to August 1998 into seven time stages by using the currency rate dynamics, the price dynamics for synthetic GKO bonds with 90-, 180-, 270-day payments periods, and the information on the RTS index variation which characterizes the Russian stock market outside the stock exchange.

Stage 1. January--May 1995

The first stage is the period in which the Russian State debt market was formed. The currency corridor introduced at the end of 1994 and some other severe measures taken to stabilize the rouble gradually made the rates in GKO--OFZ market very attractive to investors (150-180% at the beginning of this stage), since they allowed high rates of currency return. In this period an important role is played by the active struggle with inflation. This struggle results in a significant decrease of this macroeconomics

factor (from 17.8% in January 1995 to 5.4% in July). At this stage the Government could evolve such rigid policy by using the stabilization credits given by the International Monetary Fund. In this period the dollar exchange rate grew from 3500 roubles per dollar till 5000 roubles per dollar.

Stage 2. June--July 1995

The second stage in the history of the Russian stock market is characterized by the final victory over inflation and a sharp decrease in the bounds of the currency corridor. It should be noted that during the summer months in 1995 the dollar exchange rate decreased till 4500 rouble per dollar (almost by 10% in contrast to May) and the inflation decreased till 2--3% per month.

Stage 3. August--December 1995

The third stage is completely determined by the bank crisis in August--September 1995 and the fact that investors were not confident in the results of the December elections to the State Parliament (Duma). To a large extent, the bank crisis was caused by low rates of interest in the State debt market, as well as by inability of several participants in the market to work when rates of interest are low and the currency market is stabilized. Some banks sold portfolios of GKO in order to maintain the current liquidity, which resulted in large oscillations in rates of interest in the State debt market. In the second half of this period, the lack of confidence of investors in the results of the Parliament elections in 1995 was a destabilizing factor. It should be noted that the absence of big players in the stock market resulted in extremely low daily trade volumes. It should also be noted that till the beginning of 1997 the scales of the stock market were significantly lower than the scales of the GKO--OFZ market. For example, with respect to such an important parameter as the daily trade volume, the stock market was more than 10 times smaller than the State debt market. It is not astonishing that the influence of the stock market on the economical life of the country at that time was insignificant.

Stage 4. January--June 1996

The fourth stage in the history of the Russian stock market is characterized by the pre-election fever in 1996. One can observe efforts to maintain the rouble exchange rate and to attract money resources necessary for successful per-election campaign. In April--May the Central Bank of the Russian Federation sold more than 3 milliards dollars from its currency reserves. These problems were solved successfully (the rouble exchange rate varies only by 6% during the entire period, i.e., from 4700 roubles per dollar to 5000 roubles per dollar), which resulted in a sharp increase in rates of interest in the Russian stock market. To the end of this period, the GKO rates grew up to the annual interest of 250—300%. At this stage, without any doubt, one had to take into account the psychological factor that the majority of players were not sure in the further continuation of reforms. It should also be noted that the response of the stock market to the pre-election fever was different. Starting from March 1996, the plot of the RTS index shows a stable-increasing trend.

Stage 5. July 1996--October 1997

This stage is characterized by the President election, the end of the Chechen war, and the credit rating given to Russia by the International agencies. The further efforts to maintain the rouble exchange rate and to decrease inflation, as well as to attain political stabilization, resulted in a further decrease in rates of interest. In new issues one could see obvious trends to increase their periods of life; for instance, year-term GKO and OFZ with constant coupons appear. Russia received regular transfers of money from the International Monetary Fund in due time. In the second half of this period, another positive factor appeared: in November 1996 Russia obtained the credit rating BB+. To the spring of 1997 the rouble rate of annual interest of GKO attained 15--18%, which corresponds to 10--14% foreign exchange income. These rates of interest are comparable with rates of interest provided by national bonds of other countries with the same credit rating. All premises for further growth of prices and bond indices at this economical stage were exhausted. Before Russia received the credit rating, the price dynamics in the stock market was rather flaccid and the daily trade volumes rather low. After Russia received the credit rating, the share quotation in the stock market grew extremely fast till the end of the period. On October 6, 1997, the RTS index attained the record-breaking value of 571.66 points, the largest value during its entire history. At this stage, a relative increase in the RTS index was almost 500% during half a year. It should be noted that in this period the number of bonds passing the listing procedure in RTS and daily trade volumes grew tremendously. To the summer in 1997, the scales of the State debt market and the stock market became comparable. The dollar exchange rate varied weakly in the entire period and attained the level of 5900 roubles per dollar to the end of the stage. In fact, the end of this stage coincides with ‘’Black Tuesday" 28 October, when the RTS index fell by 28%.

Stage 6. November 1997--March 1998

The world financial crisis attacked, to some extent, the markets in all countries over the world. In a relatively short period of time, the bond indices fell by 10--15%, while the RTS index dropped almost by one third. Attempts to stabilize the situation by maintaining the rouble exchange rate and increasing the rates of interest had only a local success and allowed a temporary stabilization in the State debt market at the beginning of March. To the beginning of March, the state of affairs in the stock market had also changed somewhat to the best, but the RTS index did not even approach the pre-crisis values. It should be noted that, in spite of the unfavorable situation in the foreign and Russian financial markets, the Government succeeded in maintaining the rouble exchange rate with respect to dollar and, on January 1, even denominated the rouble one thousand times. At the end of this stage, a political crisis struck Russia: the cabinet of Victor Chernomyrdin was forced to resign. At the same time the confrontation among the branches of the Government reinforced. In the middle of March the rating agencies decreased the credit rating of Russia. The prices in the world oil market began to fall violently, which decreased the income of the most of Russian oil companies and sharply decreased the returns obtained in foreign trade.

Stage 7. April--August 1998

The seventh stage is the period of the next collapse in the stock market. Efforts to maintain the rouble exchange rate and an increase in the rate of interest did not lead to a cardinal solution of the problems that appeared at the previous stage. The currency outflow from Russia got out of control and the pressure on the Central Bank grew violently. An actual threat of the rouble devaluation makes investors to sell their GKO portfolios in a hurry at dumping prices and to buy the currency for exportation. The positive feedback circuit is thereby closed. To the end of July the process got out of control of the Government completely: there were no new GKO trading, the payments for the old issues were covered directly from the budget, the rouble exchange rate was maintained by selling gold and currency reserves of the Central Bank. The oil crisis significantly restricted the possibilities of the Central Bank somehow to enlarge its gold and currency reserves by buying the currency from exporters. As a result, the Government defaulted on payments for GKO-OFZ during 1998--1999 years and declared a 90-day moratorium on foreign debts (this was an actual default); a large-scale rouble devaluation occurred. All this led to a system crisis in the entire country. Although the stock market continued to operate after August 17, the liquidity was lost to a large extent. To the end of August the prices of the majority of bonds attained their minimum and the daily trade volume decreased by several tens of times.

Tables 2-3 show some general characteristics of the internal State debt market (Table 2) and the stock (Table 3) during the period 1995-1998 within the framework of the stages listed above.

Table 2.

Stage number	Stage bounds (duration in days)	Debt volume to the end of the stage (billion denom. Roubles)	Variation in synthetic bond price	Synthetic bond rate of interest (averaged over the period)
1	January 1,1995- May 31,1995 (150)	-	28.7% (for 90-day bonds) 58.9% (for 180-day bonds)	126.4% (for 90-day bonds 122.% (for 180-day bonds)
2	June 1, 1995- July 31,1996 (61)	-	-8.7% (for 90-day bonds -6.5% (for 180-day bonds	55.8% (for 90-day bonds) 68.7% (for 180-day bonds)
3	August 1.1995-	76.7	0.4% (for 90-	94.0% (for 90-

	December 17,1995 (139)		day bonds) -9.6% (for 180-day bonds)	day bonds) 114.0% (for 180-day bonds)
4	December 18,1995-June 30, 1996 (195)	-	1.4% (for 90-day bonds) 1.7% (for 180-day bonds)	92.3% (for 90-day bonds) 111.2% (for 180-day bonds)
5	July 1, 1996-October 31,1997 (489)	-	17.9% (for 90-day bonds) 37.5% (for 180-day bonds) 24.7% (for 270-day bonds)	30.8% (for 90-day bonds) 35.1% (for 180-day bonds) 25.9% (for 270-day bonds)
6	November 1,1997-March 20, 1998 (140)	415.7	-0.9% (for 90-day bonds) -3.2% (for 180-day bonds) -5.8% (for 270-day bonds)	21.6% (for 90-day bonds) 29.5% (for 180-day bonds) 30.5% (for 270-day bonds)
7	March 21, 1998-August 14,1998 (146)	387.1	-24.6% (for 90-day bonds) -36.6% (for 180-day bonds) -44.7% (for 270-day bonds)	50.2% (for 90-day bonds) 53.5% (for 180-day bonds) 52.5% (for 270-day bonds)

Table 3.

Stage number	Stage bounds (duration in days)	Stock market capitalization at the beginning of the stage	RTS index variation.	Stage-averaged day trading volume in the RTS system
3	September 1, 995	18.0 mill. Doll.	-17.6%	2.37 mill. Doll.

	December 7,1995 (108)			
4	December 8,1995 June 30, 1996 (195)	14.8 mill. Doll.	151.9%	8.77 mill. Doll.
5	July 1, 1996 October 31, 1997 (489)	37.2 mill. Doll.	106.8%	40.01 mill. Doll.
6	November 1,1997 March 20, 1998 (140)	76.6 mill. Doll.	-20.5%	67.08 mill. Doll.
7	March 21, 1998 August 14, 1998 (146)	60.9 mill. Doll.	-65.7%	45.83 mill. Doll.
On August 14, 1998 the stock market capitalization was 20.9 mill. Doll.				

How can one explain the five-times increase in the price of the Russian stocks, while the general macroeconomics factors in Russia demonstrate at most only a tendency for a decrease in the rate of recession?

Even a cursory examination of Fig. 1 shows that there exists a high correlation between the dynamics of gold and currency reserves of the Central Bank and the dynamics of the RTS index in the period of its growth from the end of 1996 till its fall after October 27, 1997. The correlation between the gold and currency reserves of the Central Bank and the RTS index, calculated over the period from January 31, 1996, to August 31, 1998, was 0.74 and the correlation calculated over the period from December 31, 1996, to August 31, 1998, was even larger, 0.86.

The RTS index in this period increased approximately by a factor of five and than fell somewhat lower than its initial level. Therefore, we can assume that the volume of money resources attracted to the Russian stock market in this period was significantly larger than that attracted in 1995 and at the beginning of 1996. Unfortunately, we do not have a possibility to receive any information on this volume. Since the stock market participants themselves did not have such information, we can assume that the data on the gold and currency reserves of the Central Bank given in mass media can be a reference point common to everybody, especially under the conditions of comparatively slowly varying fundamental money factors (Figs. 2 and 3 in Appendix 1).

Generally speaking, in the period from 1995 to the present time (which is not a good time for the Russian stock market), one can observe the following two stable characteristic features. The first is that the majority of the market participants believe that the stocks of many Russian companies are underestimated. Different agencies and broker companies gave different estimates, but most of them confirm the fact that the prices of Russian stocks are significantly lower than the prices of western analogs. As is known, the common expectation of the market participants necessarily manifests itself in some way. The second characteristic feature of Russian stocks is that the dividends of ordinary stocks are extremely low if paid at all. To our knowledge, if for the stock price we take the stock price on September 1, 1998, and for the dividends we take the dividend magnitude in 1997, then the ratio *D/P* is approximately equal to 0.04 (this corresponds to the data given in the journal «Expert»). If we compare the 4-percent (with respect to the low price of the stock) annual dividends with the existing refunding rate approximately equal to the annual interest of 20--40%, then we can see that the purchase of stocks of Russian companies by nonstrategic investors is in character a game intended to increase the stock price. (The functioning of the futures market of the most liquid Russian stocks allowed additional speculative games.)

Just this second «peculiar» feature of the Russian stock market denudes the stock course price of some natural lower bound. The price can fall to a very low level. Besides of this, this fact practically annihilates any correlation between the stock course price and the actual situation in the corresponding industry branch.

However, here we must stress the following fact. If a company in the Russian stock market becomes attractive to foreign investors, then the market reacts to this fact immediately.

So, we can hypothesize that the Russian stock market is very attractive to foreign investors from the viewpoint of games intended to increase the stock price.

The infrastructure of the Russian stock market is well organized (more exactly, the infrastructure can readily be reconstructed if money resources are available) but ... so far the risks of operations in Russia are very high. If the risk decreases (provided that the problem of the debt reconstruction is solved, the foreign stabilization loans are given, and political stability is attained), then the stock market can regain its former liquidity in a very short period of time.

Although there are such striking peculiar features, the Russian stock market is not completely isolated, first of all, because of foreign investors.

As we can see in Fig. 1, these time-series have brightly expressed trends and, according to the information in Table 3, these trends are highly correlated in several pairs. It is natural to study the problem of cointegration of these time series. But a statistical analysis of time series of first differences of the variables under study (see Fig. 2) allows us to conclude that these series are nonstationary. Therefore, we cannot say that these series are cointegrated, at least, in the classical sense.

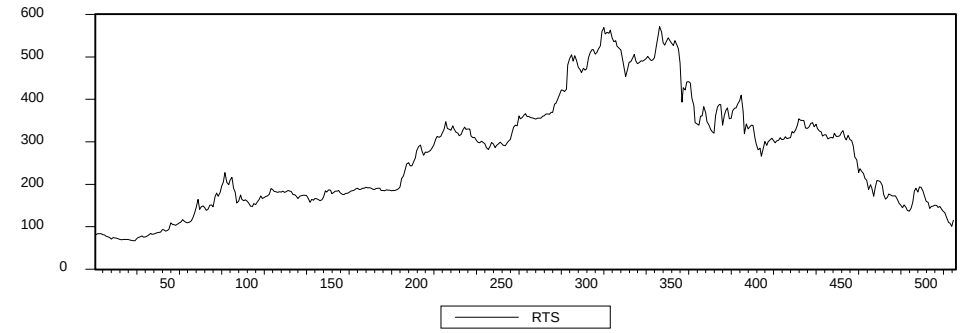
Is the time dependence between the stock index, the gold and currency reserves, and the state bond prices a special characteristic feature of the Russian financial market? Studying the dynamics of stock markets in different countries during the last three years, we see that not only in the Russian market one could observe a five-times increase in the stock index followed by its drop to the original level. During 1996-98 the Venezuelan stock market varied in a similar way, and moreover, these changes were accompanied by variations in the gold and currency reserves just as it happened in Russian.

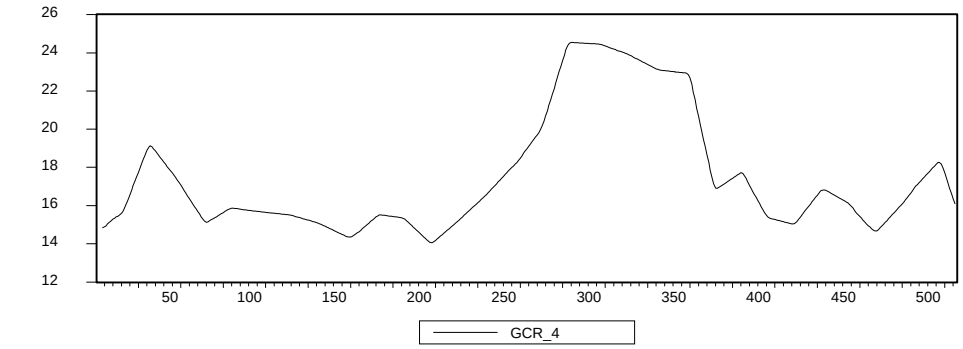
Является ли наблюдаемая зависимость между фондовым индексом,

Table 4. Correlation matrix

	RTS	GKPR_180	GKPR_90	GCR_4
RTS	1.000000	0.802665	0.730466	0.733617
GKPR_180	0.802665	1.000000	0.972752	0.429397
GKPR_90	0.730466	0.972752	1.000000	0.407460
GCR_4	0.733617	0.429397	0.407460	1.000000

Fig. 1 Time dependence of the RTS, GKPR_180, GKPR_90, GCR_4 series





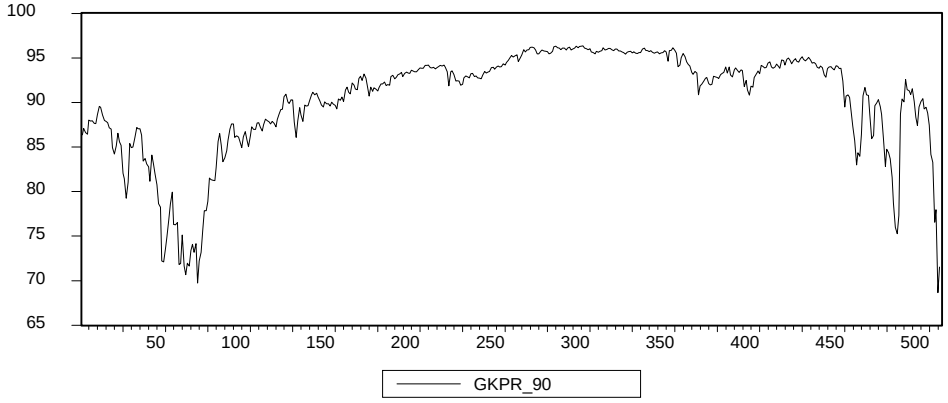
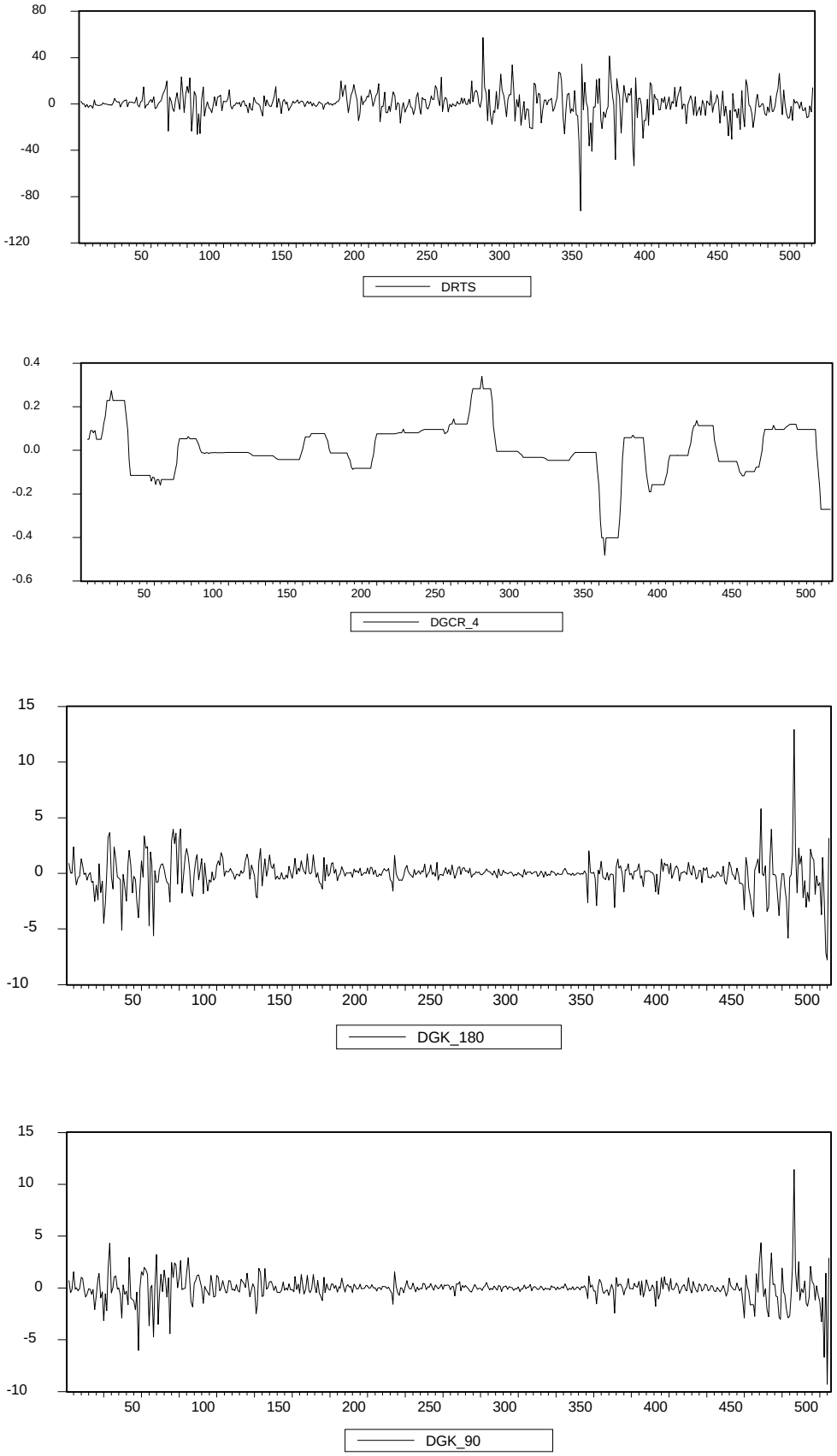


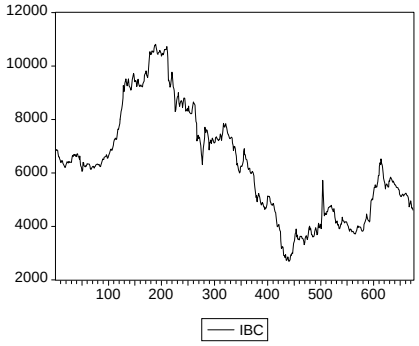
Fig. 2 Time dependence of differences

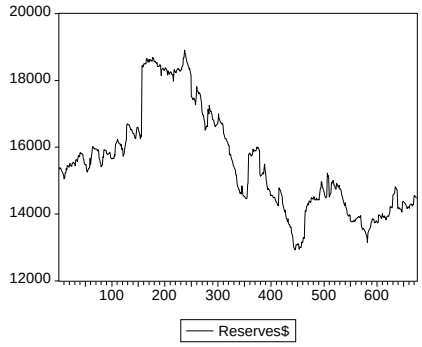


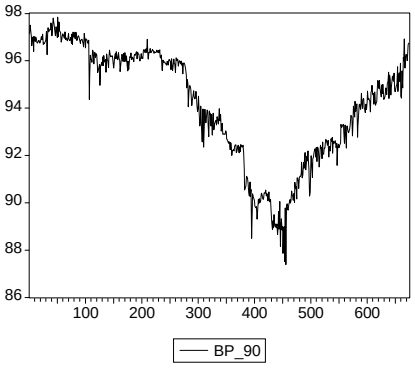
An analogy with the Russian stock market can be observed for the following time series of the Venezuelan market:

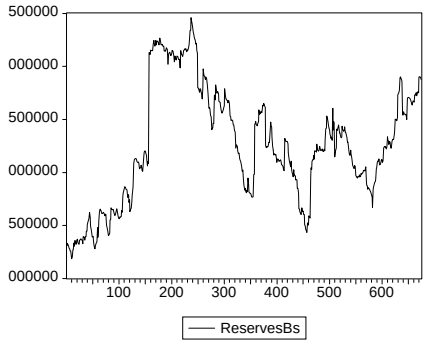
IBC-stock index, Reseves$ - international reserves in mill. Doll., ResevesBs - international reserves in bolivars, BP_30, BP_60, BP_90 - the prices of synthetic state bonds with nominal value of 100 and with 30-, 60-, 90-day periods of payment. The time dependence of the series IBC and Reserves$, BP_30 and BP_60 and BP_90, DReserves$ and DReservesBs during 2.5 years from January 2, 1997, to August 5, 1999, is shown in Fig. 3. One can readily see that these series have distinct trends and the correlation between IBC and ReservesBs is very large. If we neglect details, then we see that the dynamics of time series has much in common for Russia and Venezuela (Figs. 1 and 3).

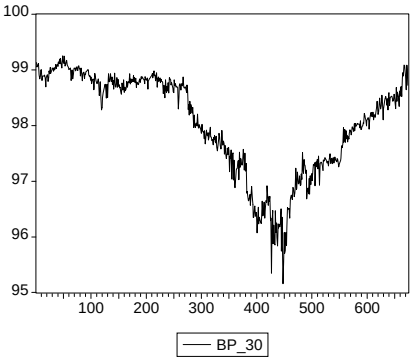
Fig. 3 Dynamics of the finanacial market in Venezuela

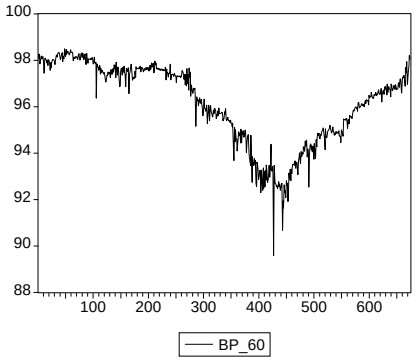












2. Arbitrage and quasi-arbitrage operations in the cash and futures markets

Futures contracts in the stock market

Securities are financial tools based on real assets. *Financial derivatives are* financial tools based on other simpler financial tools. It is commonly agreed to classify forward and futures contracts, options, financial futures and financial futures options as financial derivatives. In turn, financial futures as divided into currency futures, interest futures (for promissory notes), and swaps and futures for stock market indices.

When the financial derivatives are bought, they impose certain obligations on the seller and the buyer concerning a fixed time instant in the future (or until a fixed time instant in the future) and the tools that underlie the financial derivatives. Transactions of such type are called *forward transactions* (transactions for a term) in contrast to transactions in the cash market (cash or spot transactions) with immediate payment and delivery. A classical example of spot transactions is buying (selling) of a *forward contracts*.

A forward contract is an agreement between two parties one of which commits oneself to buy and the other party commits oneself to sell a certain amount of goods or securities at a specified price at a future time instant fixed in the agreement (on the day at which the contract is executed). In this case neither money nor goods are required till the time indicated in the contract. Since there are no standards for terms and amounts of goods, forward contracts are used only out of stock markets.

{Strictly speaking, according to this definition, goods forwards and futures cannot be classified as financial derivatives, since they are based on actual goods but not on financial tools. However, in the literature, this specific characteristic of goods forwards (futures) is not taken into account when possible combinations of financial derivatives and real assets are analyzed.}

Futures contracts are traded in stock markets. A difference between futures and forward contracts is that the delivery amount of certain goods is fixed in the futures contract (both in nomenclature and in quality) and futures contracts must be executed in certain fixed months of the year. In this case, the stock market guarantees that the contract will be executed and, in this sense, it is absolutely unimportant for the buyer (seller) who, in fact, executes the contract (i.e., who delivers the goods or securities and who pays for the delivery). In the stock market jargon, *to buy a futures contract* means *to take a long position* (*to go long*) and *to sell a futures contract* means *to take a short position* (*to go short*). If a participant of the futures market simultaneously goes long and short in the same futures contract, it is said that *the position* of this participant *in this futures contract is closed*.

To fulfill the obligations assumed, the stock market obliges both the seller and the buyer of futures to deposit some money resources (the initial margin) in the Settlement House of the market. This initial margin will be returned when the contract is executed or the position is closed. The quantity of the initial margin depends on the concrete assets and is determined by the market with account of the volatility of the price of the assets.

In the Russian market the quantity of the initial margin is usually equal to 5% of the price of the contract. In some markets it is allowed to deposit up to 70% of the initial margin in liquid securities indicated in advance.

Besides of the initial margin deposited before a position is taken, the market obliges all participants with open positions to deposit some additional money to compensate the negative (from the viewpoint of those in open positions) daily variation of the price of the contract, i.e., the so-called negative variation margin. For this purpose, every day the

calculated (settlement) price is announced in the market. This price is calculated over the results of the daily auction (often, it is simply the closure price, i.e., the price during the last minutes of the auction). Then the daily variation in the calculated price (the settlement change) is announced as the difference between the calculated prices of the current and preceding days. On the basis of these data, the Settlement House of the market reestimates positions of the participants, i.e., performs marking them to the market.

If, in this case, it turns out that the settlement change of the contract is positive, there are the following consequences:

- the variation margin is positive for those who go long in this contract (the sum equal to the settlement change will be paid into his current account);
- the variation margin is negative for those who go short in this contract (the sum equal to the settlement change will be taken from his current account). If the settlement change of the contract turns out to be negative, then the situation is opposite: those who go long «lose», while those who go short «win». Thus the absolute value of the variation margin is equal to the absolute value of the variation in the current account corresponding to the open position, while these variations are of opposite signs for the long and short positions.

The money in the current accounts of the contract buyers and sellers in the Settlement House in the form of the initial margin provide the possibility for the market to cover losses of both parties on any day if the prices volatility is unfavorable (the variation margin is negative). At the end of each trading day (before the next trading day starts), both the buyer and the seller must guarantee that their current accounts in the Settlement House contain money not less than a certain sum. This sum is called the *maintenance margin.* As, because of a negative daily variation margin, the sum of money in the current account decreases till the value of the maintenance margin (or becomes less than this value), the seller (buyer) of the contract must add to his current account the amount of the initial margin. If this requirement is violated, the Exchange on its own may close this unprovided position at the beginning of the next trading day.

Basic parameters that provide a gain in arbitrage transactions

Arbitrage transaction is a set of operations in the market that bring an income because of the difference in prices existing at the moment in one or several markets. Arbitrage transactions without use of the ownership initial capital are often called *pure arbitrage*. Under certain conditions an arbitrage transaction can turn into pure arbitrage.

As an example, we consider the situation in the spot market of securities and in the market of borrowed capital. If only the tools of these markets are used, then the income obtained by sale and purchase operations depends on the future price of the assets. Since arbitrators do not know the future dynamics of prices, one can speak about each arbitrage transaction with some probability (if we mean the future) or as of a lost possibility (if we mean the past). Hence we can treat each transaction not as an arbitrage but as a speculative transaction.

A more definite situation arises if there exists both cash and forward (futures) markets for a certain type of goods. To analyze a typical situation in the cash, futures, and money markets, we need the following notation. For the unit of goods we take 100 stocks of a company whose dealer sale and purchase prices at the initial time t are equal to $P^a(t)$ and $P^b(t)$, respectively, ($P^a(t)>P^b(t)$) and whose corresponding prices of execution of futures contracts (at time T) are

$F^a_T(t)$ (for the buyer of the contract) and
$F^b_T(t)$ (for the seller of the contract) $(F^a_T(t) \geq F^b_T(t))$.

We assume that the annual borrowing rate possible for the arbitrator is $r_{bor}(t)$ and the possible lending rate is $r_{len}(t)$. Moreover, we assume that

a) the transaction costs for buying 100 stocks of a company are equal to the fraction r_{tr} of the purchase price;

b) the loan of 100 stocks of a company for the term $(T-t)$ is equal to the fraction ρ_{loan} of the sale price at the time of signing the loan contract;

c) the initial margin in the futures market is equal to the fraction r_m of the contract price.

By using the above notation, one can easily analyze the payment flows arising in arbitrage transactions of two types (the direct and reverse cash-and-carry strategies).

Description of the direct arbitrage operation (cash-and-carry strategy)

At the current time t

1) To take the credit $B=P^a(t)*(1+r_{tr})+F^b_T(t)* r_m$ for the period $T-t$ at the $100*r_{bor}(t)$ annual rate.

2) To buy 100 stock of a company in the cash market at the price $P^a(t)$ and to pay the transaction costs $P^a(t)* r_{tr}$..

3) To sell the futures contract (going short) for delivery of 100 stocks of the company after the time interval $T-t$ at the price $F^b_T(t)$ and to deposit the initial margin $F^b_T(t)* r_m$

At time T of the contract execution

1) To deliver 100 stocks of the company according to the futures contract. To receive, according to the contract, the sum of money $F^b_T(t)$ and the initial margin $F^b_T(t)* r_m$.

2) To return the sum of the money loan which together with the interest is

$$[P^a(t)*(1+r_{tr})+ F^b_T(t)* r_m]*(1+r_{bor}(t)*(T-t)).$$

The gain received during the time interval $T-t$ is equal to

$$\begin{aligned}\Pi_1 &= F_T^b(t)*(1+r_m)-[P^a(t)*(1+r_{tr})+F_T^b(t)*r_m]*(1+r_{bor}(t)*(T-t))= \\ &= F_T^b(t)*(1-r_m*r_{bor}(t)*(T-t))- \\ &P^a(t)*[1+r_{bor}(t)*(T-t)+r_{tr}+r_{tr}*r_{bor}(t)*(T-t)]\end{aligned} \qquad (2.1)$$

The condition $\Pi_1 > 0$ can be written in the form

$$r_a(t,T-t) > UBpa(t,T-t) \tag{2.2}$$

where the direct arbitrage interest rate $r_a(t,T-t)$ is defined by the relation

$$r_a(t,T-t) = \frac{F_T^b(t) - P^a(t)}{P^a(t)} * \frac{1}{T-t}, \tag{2.3}$$

and the symbol $UBpa(t,T-t)$ stands for the Upper Bound for pure arbitrage

$$UBpa(t,T-t) = r_{bor}(t) * [1 + r_{tr} + \frac{r_{tr}}{r_{bor}(t)} * \frac{1}{T-t} + \frac{F_T^b(t)}{P^a(t)} * r_m] \tag{2.4}$$

Description of the reverse arbitrage operation (reverse cash-and-carry strategy)

At the current time t

1) To loan 100 stocks of a company for the period *T-t*.
2) To sell 100 stocks of the company in the cash market at the price $P^b(t)$ and to pay the transaction costs $P^b(t)* r_{tr}$.
3) To buy a futures contract (to go long) for delivery of 100 stocks of the company after the time interval *T-t* at the price $F^a_T(t)$ and to deposit the initial margin $F^a_T(t)* r_m$ in the Settlement House of the market.
4) To deposit the remaining sum of money $L(t)= P^b(t)*(1-r_{tr})- F^a_T(t)* r_m$ for the time interval *T-t* at the rate of $100* r_{len}(t)$ annual return.

{It is assumed that $L(t)>0$, which, in turn, bounds the value $F^a_T(t)$: $F^a_T(t)<P^b(t)*(1-r_{tr})/r_m$. Since the variables r_m and r_{tr} are small ($r_m<0.1$ and $r_{tr}<0.1$), this inequality holds practically always.}

At time T of the contract execution

1) To receive the sum $[P^b(t)*(1-r_{tr}) - F^a_T(t)* r_m](1+r_{len}(t)*(T-t))$ of money deposited.
2) To receive the delivered 100 stocks of the company according to the futures contract and, according to the contract, to pay the sum of money $F^b_T(t)$ and to receive the initial margin $F^b_T(t)* r_m$.
3) To return 100 stocks and to pay the sum $P^b(t)* \rho_{loan}$ for the money loan.

{In transaction 2, the combination of operations 1) and 2) at time *t* and operation 3) at time *T* is called the *short sale* or the *sale without settlement*. This concerns any possible assets, not only god. However, such an operation is not always possible because of various reasons.}

The gain received during the time interval *T-t* is equal to

$$\Pi_2 = L(t)*(1+r_{len}(t)*(T-t)) + F_T^a(t)*r_m - F_T^a(t) - P^b(t)*\rho_{loan} =$$
$$= P^b(t)*[1+r_{len}(t)*(T-t) - r_{tr} - \rho_{loan} - r_{tr}*r_{len}(t)*(T-t)] \qquad (2.5)$$
$$- F_T^a(t)*(1+r_m*r_{len}(t)*(T-t))$$

The condition $\Pi_2 > 0$ can be written in the form

$$r_a'(t,T-t) = \frac{F_T^a(t) - P^b(t)}{P^b(t)} * \frac{1}{T-t} < LBpa(t,T-t),$$
$$LBpa(t,T-t) = r_{len}(t)*[1 - r_{tr} - \frac{r_{tr}}{r_{len}(t)} * \frac{1}{T-t} - \frac{\rho_{loan}}{r_{len}(t)} * \frac{1}{T-t} - \frac{F_T^a(t)}{P^b(t)} * r_m] \qquad (2.6)$$

Here, by analogy with (2.3) and (2.4), we use the following notation: $r_a'(t,T-t)$ stands for the reverse arbitrage interest rate and $LBpa(t,T-t)$ for the Lower Bound of pure arbitrage.

Let us note the following characteristic features of arbitrage strategies:

1. The cash-and-carry strategies considered do not use the initial ownership capital. Hence the usual notion of gain cannot be used for characterizing such cash-and-carry strategies. However, the *norms of income of the direct and reverse arbitrage operations* defined by relations (2.3) and (2.6) characterize the relation between the futures and spot markets under existing loan and deposit rates.

2. It is unprofitable to carry out the direct and reverse cash-and-carry strategies simultaneously, since we have

$$\Pi_1 + \Pi_2 < 0,$$

in view of the inequalities $F^a_T(t)-F^b_T(t)>0$, $P^a(t)-P^b(t)>0$, $r_{bor}-r_{len}>0$.

3. The assumption that it is impossible to realized a pure arbitrage transaction implies the following natural two-sided bounds on the prices of futures contracts:

$$P^b(t)*(1+(T-t)*LBpa(t)) < F_T^a(t) \approx F_T^b(t) < P^a(t)*(1+(T-t)*UBpa(t)) \quad (2.7)$$

4. If we neglect the spread between the purchase and sell prices, then we have $r_a'(t,T-t) \approx r_a(t,T-t)$. In this case the two-sided inequality (2.7) turns into the following two-sided inequality for the norm of income of the arbitrage operation: the direct and reverse cash-and-carry strategies of pure arbitrage carried out at time t during the period T-t are unprofitable if

$$LBpa(t,T-t) < r_a(t,T-t) < UBpa(t,T-t) \qquad (2.8)$$

In other words, a situation, which theoretically is arbitrageless, can exist only if inequality (2.8) is satisfied.

Synthetic bond and quasi-arbitrage

In the preceding section we studied the situation in which it was possible to realize the pure arbitrage. However, if the ownership capital exists, a transaction of cash-and-carry type can be regarded as an alternative to riskless investments in Treasury bills. Under certain conditions, this alternative transaction is more profitable than investment in the state security bonds. A realization of this possibility is called *quasi-arbitrage*.

Let us study this possibility in more detail. We assume that an investor does not want to loan money and is ready to spend the sum $B=P^a(t)*(1+r_{tr})+F^b_T(t)*r_m$ of his own money in order to realize a direct arbitrage transaction. Then the following two operations are excluded from the cash-and-carry strategy: one need not loan money and return this money with interest .

We see that, for the operation under study, the payment flow is equivalent to the payment flow in the case of purchasing a discount couponless bond and possessing this bond till its repayment. Such a bond, which does not exist in nature, is called a *synthetic bond*. In other words, a synthetic bond is a set of operations with financial tools whose payment flow is equivalent to the payment flow of a couponless bond. One can define a synthetic bond by the following symbolic relation:

Synthetic T-bill = Spot-Futures

For the initial costs of size *B(t),* the profit obtained by the investor is equal to

$$\overline{\Pi_1} = F_T^b(t) - P^a(t)*[1+r_{tr}],$$

and the income of this transaction (in other words, the lending rate on the synthetic bond market calculated by the simplest interest formula with one-year time base) is equal to

$$\tilde{r}(t,T-t) = \frac{\overline{\Pi}_1}{B(t)*(T-t)} = \frac{F_T^b(t) - P^a(t)*(1+r_{tr})}{[P^a(t)*(1+r_{tr}) + F_T^b(t)*r_m]*(T-t)}. \qquad (2.9)$$

Suppose that, at the same time, the lending rate on the GKO market with the payment period *T-t* is equal to $\tilde{r}_{len}(t,T-t)$. *Then, by definition, the condition*

$$\tilde{r}(t,T-t) > \tilde{r}_{len}(t,T-t) \qquad (2.10)$$

is a condition under which the quasi-arbitrage is possible for this investor. In this case, the price of the futures contract $F^b_T(t)$ must satisfy the inequality

$$F_T^b(t)*(1-r_m*\tilde{r}_{len}*(T-t)) > P^a(t)*[1+r_{tr}+\tilde{r}_{len}(t)*(T-t)+r_{tr}*\tilde{r}_{len}(t)*(T-t)].$$

By analogy with $UBpa(t,T-t)$, we define the upper bound of the quasi-arbitrage operation as follows:

$$Ubqa(t,T-t) = \tilde{r}_{len}(t,T-t)*[1+r_{tr}+\frac{r_{tr}}{\tilde{r}_{len}(t,T-t)}*\frac{1}{T-t}+\frac{F_T^b(t)}{P^a(t)}*r_m]. \qquad (2.11)$$

Then we can easily rewrite (2.9) as

$$\tilde{r}(t,T-t)=\tilde{r}_{len}(t,T-t)*\frac{r_a(t,T-t)-r_{tr}/(T-t)}{Ubqa(t,T-t)-r_{tr}/(T-t)}\ . \qquad (2.12)$$

Hence the quasi-arbitrage realization condition (2.10) acquires the form

$$r_a(t,T-t)>Ubqa(t,T-t)\,. \qquad (2.13)$$

If we compare the opposite inequality, which means that the quasi-arbitrage is impossible, with the inequality $\Pi_1<0$, which means that the pure arbitrage is impossible, then we can easily see that the only difference between them is that r_{bor} is replaced by $\tilde{r}_{len}$. If $r_{bor}>\tilde{r}_{len}$, then $Ubqa(t,T-t)>UBpa(t,T-t)$. However, for a more realistic condition $0<1-\tilde{r}_{len}/r_{bor}<<1$, the following inequality is satisfied: $Ubqa(t,T-t)<UBpa(t,T-t)$, i.e., the condition that the pure arbitrage is impossible is more rigid than that for the quasi-arbitrage. This means that even if the pure arbitrage is impossible in the market (for investors who do not have their own capital), a realization of quasi-arbitrage is possible for investors who have their own initial capital. Recall that the initial finance state (the bench mark) of an investor realizing the quasi-arbitrage is different than that of an investor realizing the pure arbitrage.

Reverse quasi-arbitrage

Assume that an investor possesses 100 stocks of a company (there exist futures contracts for these stocks) and a sum of money $B(t)=F_T^a(t)*r_m+P^b(t)*r_{tr}$. He sells 100 stocks at the price $P^b(t)$, pays the transaction costs $P^b(t)*r_{tr}$, and buys a futures contract for 100 stocks of this company with payment period T. For this futures contract he pays the initial margin $F_T^a(t)*r_m$. Then the investor deposits the money obtained for the stocks at the rate of $100*\ r_{len}(t)$ annual return and thus receives the sum $P^b(t)*(1+r_{len}(t)*(T-t))$ on the day of the contract execution. He pays the price of the contract $F_T^a(t)$ from the money obtained and gets the 100 stocks back.

As the result of these operations, the investor has the same 100 stocks plus the money profit

$$\overline{\Pi_2}=P^b(t)*(1+r_{len}(t)*(T-t))-F_T^a(t)-P^b(t)*r_{tr} \qquad (2.14)$$

The rate of return of the above operation can be calculated as the ratio of the profit (2.14) to the cash expenditures divided by the time interval between the beginning and the end of the operation:

$$\hat{r}(t,T-t)=\frac{\overline{\Pi_2}}{B(t)*(t-t)}=\frac{P^b(t)*[1+r_{len}(t)*(T-t)-r_{tr}]-F_T^a(t)}{[F_T^a(t)*r_m+P^b(t)*r_{tr}]*(T-t)} \qquad (2.15)$$

By analogy with (2.10), the following inequality is a condition that a quasi-arbitrage realization does not lead to losses for the investor:

$$\hat{r}(t,T-t) > \tilde{r}'_{len}(t,T-t)\,. \qquad (2.16)$$

By using notation (2.6) and taking into account (2.15), we can rewrite condition (2.16) in the form

$$r'_a(t,T-t) < Lbqa(t,T-t),$$
$$Lbqa(t,T-t) = r_{len}(t)*[1-r_{tr} - \frac{r_{tr}}{r_{len}(t)*(T-t)} - \frac{F_T^a(t)}{P^b(t)}*r_m] - \qquad (2.17)$$
$$[\tilde{r}'_{len}(t,T-t) - r_{ltn}(t)]*(r_{tr} + \frac{F_T^a(t)}{P^b(t)}*r_m)$$

If the spread between the purchase and sell prices is neglected, then we have $r'_a(t,T-t) \approx r_a(t,T-t)$. In this case the condition that both the direct and the reverse quasi-arbitrage is impossible can be written in terms of a two-sided inequality for the norm of income of the arbitrage operation as follows:

$$Lbqa(t,T-t) < r_a(t,T-t) < Ubqa(t,T-t) \qquad (2.18)$$

For some natural relations between the existing rates of return, inequality (2.18) implies inequality (2.8). But inequality (2.8) does not necessarily implies (2.18).

Norm of income of arbitrage operations as an indicator of arbitrage possibilities

It follows from the above theoretical arguments that one can estimate the possibility of an arbitrage strategy by comparing the norm of income of the arbitrage operation $r_a(t,T-t)$ with the bounds $UBpa(t,T-t)$, $Ubqa(t,T-t)$, $Lbqa(t,T-t)$, $LBpa(t,T-t)$. Let us consider the case most frequently encountered in practice, i.e., the case of embedded inequalities

$$LBpa(t,T-t) < Lbqa(t,T-t) < Ubqa(t,T-t) < UBpa(t,T-t)$$

for each *t* and *T-t*. All possible arbitrage realizations are summarized in Table Z, which is rather simple.

Table Z

Relation	Profitable strategy
$r_a > UBpa$	Both the pure direct arbitrage and the direct quasi-

	arbitrage are possible.
$Ubqa < r_a < UBpa$	The rate of return of the direct quasi-arbitrage is larger than that of the GKO repayment with the same duration period
$Lbqa < r_a < Ubqa$	Neither of the above strategies is profitable
$LBpa < r_a < Lbqa$	The rate of return of the reverse quasi-arbitrage is larger than that of the GKO repayment with the same duration period
$r_a < LBpa$	Both the pure reverse arbitrage and the reverse quasi-arbitrage are possible

Risks in arbitrage operations

In the arbitrage operations studied above, there are no risks related to future price variations in the market. In this sense, such operations are riskless. However, investors who use the market financial tools depend on whether the stock market fulfills its obligations. In the second half of the 90ies the young Russian market acted under very complicated political and economical conditions. In August 1998, the GKO market, the most reliable market during the last 5 years, crashed down. Previously, in May 1998, the largest futures market in the Russian stock market collapsed. The cause was very simple: the stock market did not fulfill its obligations to preserve the initial and the variation margin but used this margin for its own game in the futures and other markets via «friendly» broker offices. Therefore, when in May 1998 it was necessary to close a lot of positions, the market could not fulfill its obligations. An attempt to solve this situation in legal form was not successful. The courts treated the futures contracts as bets and refused to consider the actions. And this was not the first time in the short Russian history of futures auctions. Hence, due to this cause, the Russian futures market in the 90ies was very speculative, although, as we show later, there were good possibilities for quasi-arbitrage operations.

3. Analysis of possibilities of arbirage operations by the example of stockes of «Lukoil» (LKOH) and RAO EES (EESR) companies

Many theories describing financial markets are based on an essential assumption that arbitrage operations in these markets are impossible. It is assumed that a market operates so that the information circulating within this market does not allow the same arbitrage possibilities to exist for a long time. In this section we show how far the situation in the Russian stock market was from this idyllic picture.

As the main object of our study, we consider the quasi-arbitrage transactions between the futures market and the spot market of stocks of Russian companies during the period from November 1996 to May 1998. Separately, we consider the possibility of pure arbitrage between these markets.

Choice of stock exchange, securities, and the time interval

During the last three years the largest trade volume of securities of Russian companies was observed outside the stock exchange market in the electronic trade system "Russian trade system" (RTS). In contrast to stock exchanges dealing with securities of at most 10-15 companies and having the daily trade volume of at most 5-7 million dollars, the RTS listing contains more than 200 securities, including practically the most liquid stocks (the so-called "blue chips"). In this case the average daily trade volume in the system exceed 20 million dollars. The number of operators (players carrying out transactions) in the RTS is considerably greater than the number of players in all Russian stock exchanges. Thus the dynamics of prices in the RTS is less affected by the momentary mood of a player or of a group of players. It should be noted that the RTS index is, de-facto, one of the most important indices of the financial activity of the country and a special mirror of the Russian stock market for Western investors, in contrast to the stock exchange indices that are known to a small number of specialists. Thus it is reasonable to take just the results of the RTS-trades as the input data for spot-transactions in our further analysis.

We consider the Russian exchange (RE) as the stock *exchange, which* provides us with data in the future market operations. First of all, this choice is stimulated by the following facts: a relatively narrow specialization of the exchange which, almost without exceptions, trades with futures for securities of Russian companies, a high daily trade volume of these transactions exceeding a similar index of other exchanges, and a large number of operators in this market. The final data for transactions in the Russian exchange were conducted on the fifteenth of each month. Types of futures contracts and the annual trade volumes related to these contracts in the Russian stock market are shown in Appendix 2.

We choose an object *of analysis*, i.e., the securities of concrete companies, so that they satisfy, at least, the following two assumptions: they must be highly liquid (i.e., their daily trade volume in these securities must be standardly high) and they must have a long market history. We believe that ordinary stocks of RAO EES and NK LUKOIL satisfy these assumptions most completely. Both these companies enter the first dozen of the largest Russian companies and the daily trade volume of their securities is comparable with the entire daily trade volume in the RTS system. The securities of these emitters were traded in the RTS system since the advent of the system of electronic trades in Russia and thus possess the largest market history.

It is necessary to take into account the fact that, in view of some reasons such as problems with reregistration of stocks in the RTS system, with money transfer, etc., futures transactions in the direct arbitrage are most frequently concluded for the next month. Hence, on each current day, we consider the possibility to use futures contracts with payment periods from 45 to 15 days.

The time series for the norm of income of an arbitrage operation $r_a(t,T-t)$, the upper bound for the pure arbitrage $UBpa(t,T-t)$, and the upper bound for the quasi-arbitrage $Ubqa(t,T-t)$ are constructed as follows. We choose a contract with payment period T_i. For all time instants t such that the variable T_i-t lies in the interval from 45 to 15 days, we calculate the values of $r_a(t,T-t)$, $UBpa(t,T-t)$, $Ubqa(t,T-t)$ by formulas (2.3), (2.4), and (2.6). Then we consider the contract for the next month with payment period T_{i+1} and, again for all times t such that T_{i+1}-t lies in the interval from 45 to 15 days, calculate the values of $r_a(t,T-t)$, $UBpa(t,T-t)$, $Ubqa(t,T-t)$. In a similar way, by formulas (2.6) and (2.17), we construct the time series for the lower bounds. Thus we obtain long time series in which we take into account the prices of all contracts with payment periods are separated from the current time instant t by 15-45 days. We shall say that such series are *series for one month contracts*.

Finally, we choose a time *interval* for our analysis taking into account the process of formation of the Russian future corporate securities market. So, the future market of NK LUKOIL securities became highly liquid in November 1996, while the future market of RAO EES securities only in May 1997. Thus the series used for further analysis start from November 1, 1996, for NK LUKOIL and from May 7, 1997, for RAO EES. Both time series end on May 25, 1998. On this day the Russian stock market ceased to operate with futures contracts.

Possibilities of practical realization for different arbitrage operations

As was already shown in Section 2, the pure arbitrage operations are profitable if the two-sided inequality (2.8) is not satisfied. Figures 1 and 2 in Appendix 3 illustrate the time series for norms of income $r_a(t,T-t)$ and the upper (*UBpa*) and lower (*LBpa*) bounds of pure arbitrage operations with LKOH and EESR stocks. The possibilities of direct quasi-arbitrage operations with the same stocks are shown in Figs. 3 and 4 in Appendix 3.

We calculated the upper and lower bounds by using the following values of parameters that are typical of the Russian stock market during this period:

$$r_{tr} \approx 0.0025, \quad r_m \approx 0.05, \quad \rho_{loan} \approx r_{len}(t)*(T-t).$$

We also used the MIBOR-30 and MIBID-30 rates as r_{bor} and r_{len} and the rate of return for GKO bonds as $\tilde{r}_{len}$.

A qualitative analysis of arbitrage possibilities for the entire time period under study is shown in Table 5. In addition to the possibilities provided by LKOH or EESR stocks, this table shows a more profitable strategy when, among two types of stocks, one can choose the most profitable on the current day (see the results in the last column).

Table 5. *Some parameters of arbitrage operations*

	NK LUKOIL	RAO EES	NK LUKOIL and RAO EES jointly
	Direct arbitrage	Direct arbitrage	Direct arbitrage
Number of observations	383	253	253
Number of observations for which the pure arbitrage is realized $r_a > UBpa$ [the number of observations in %]	168 [44%]	142 [56%]	161 [64%]
Number of observations for which the direct quasi-arbitrage is realized $r_a > Ubqa$ [the number of observations in %]	241 [63%]	177 [70%]	203 [80%]
Average rate of return $\tilde{r}$ of all direct quasi-arbitrage operations over the period (annual return in %)	30.5%	29.9%	33.2%
Average rate of return [standard deviation] $\tilde{r}$ of the direct quasi-arbitrage when its rate of return exceeds that of GKO bonds (annual return in %)	42.5% [24.8%]	38.9% [22.3%]	41.4% [26.1%]
Maximal [minimal] rate of return $\tilde{r}$ of the direct quasi-arbitrage when its rate of return exceeds that of GKO bonds (annual return in %)	333% [15.8%]	274% [14.7%]	333% [14.7%]
Average rate of return of GKO bonds [standard deviation] over the period* (annual return in %)	24.0% [9.6%]	22.1% [10.6%]	22.1% [10.6%]
Maximal [minimal] rate of return of GKO bonds over the period* (annual return in %)	104.2% [7.5%]	104.2% [9.5%]	104.2% [9.5%]

Average rate of return of MIBOR-30 [MIBID-30] over the period (annual return in %)	32.4% [21%]	29.8% [18%]	29.8% [18%]
	Reverse arbitrage	Reverse arbitrage	Reverse arbitrage
Number of observations for which the reverse pure arbitrage is realized $r_a < LBpa$ [the number of observations in %]	24 [6%]	12 [5%]	30 [12%]
Number of observations for which the reverse quasi-arbitrage is realized $r_a < LBqa$ [the number of observations in %]	78 [20%]	34 [13%]	72 [28%]

*- bonds with the same period of payment

One can easily see that the theoretical possibility of pure arbitrage during the time period in question was rather large: 44% of operations with NK LUKOIL stocks, 56% of operations with RAO EES stocks, and 64% of operations when the most profitable stocks were chosen. The theoretical possibility for reverse pure arbitrage was rather seldom. Moreover, it should be noted that, in practice, these operations in their classical form can hardly be realized in the Russian stock market, since during one day it is impossible to receive the money for the sold stocks, to buy a contract, and to deposit the rest of money. Hence the lower bound $LBpa$ is of a rather theoretical interest. As for direct and reverse quasi-arbitrage operations, in practice they were realized by some broker firms and realized successfully.

This fact is not surprising, since the number of observations in which the rate of return of direct arbitrage exceeded that of GKO bonds with the same period of payment varied, depending on the type of operation, from 63% to 80% of the total number of observations. In this case the average rate of return of these operations was approximately two times greater than the rate of return obtained from operations with GKO bonds.

However, the most of the above possibilities were realized in the market during the 5th and 6^{th} periods (see the classification given in Section 1). Table 6 shows the number of cases (in % of the total number of observations during the corresponding stage) in which it was possible to realize some arbitrage strategy (according to Table 5) while working with NK LUKOIL stocks. Table 7 shows the same data for RAO EES stocks.

Table 6

LKOH	$r_a > UBpa$	$r_a > Ubqa$	$r_a < LBqa$	$r_a < LBpa$	Pure arbitrage is unprofitable	Quasi-arbitrage is not effective
5 stage	46%	52%	25%	9%	45%	13%
6 stage	50%	76%	11%	2%	48%	13%
7 stage	16%	34%	18%	3%	81%	48%
Over all stages	44%	63%	20%	6%	50%	31%

Table 7

EESR	$r_a > UBpa$	$r_a > Ubqa$	$r_a < LBqa$	$r_a < LBpa$	Pure arbitrage is unprofitable	Quasi-arbitrage is not effective
5 stage	65%	76%	14%	7%	25%	10%
6 stage	60%	75%	9%	2%	38%	16%
7 stage	18%	39%	24%	5%	77%	37%
Over all stages	56%	70%	13%	5%	39%	17%

By comparing the data for the stages given in the tables, one can see that the behavior of the market participants at the 7^{th} stage is principally different from that at the preceding two stages. Namely, the possibility to realize effective arbitrage strategies has sharply decreased. In fact, already the two months before the futures markets collapsed and hence the four month before the default, the situation in the futures market with the two most liquid stocks illustrated the fears of participants concerning the future of the stock market.

Time structure of the norm of income for direct arbitrage operations

Usually, theorists consider increases in the rate of payment return with period of payment as a normal but not necessary phenomenon in the bond market. What situation was in the GKO market during the period from November 1996 to May 1998 and what can be said about the dependence of the norm of $r_a(t, T-t)$ and the rate of return of direct arbitrage operations $\tilde{r}(t, T-t)$ on the duration of futures contracts *T-t* in this period of time?

Let us formulate the basic hypothesis, which must be either supported or rejected by statistical data: if the period of operation $n = 365*(T-t)$ increases, then the rate of return of GKO payment $\tilde{r}_{len}(t, T-t)$, the norm of income $r_a(t, T-t)$, and the rate of return of direct arbitrage $\tilde{r}(t, T-t)$ increase.

Analyzing statistical data, we arrive at the following results. For the norm of income with payment period *n*, for $n \in [6,120]$ days, the above hypothesis holds. The linear regression obtained by the method of least squares implies the following relations (the values of standard deviationis for the regression coefficients are given in parentheses):

for NK LUKOIL stocks

r_a=0.2486+0.0024*n,
(0.225) (0.0004) R^2=0.03,
(0.226)

for RAO EES stocks

r_a=0.206+0.0036*n,
(0.046) (0.0010) R^2=0.02.

However, if *n* varies from 15 to 45 days, then there is no statistically significant dependence between the above variables and the contract duration *n*. These facts can be interpreted as follows. Participants of the futures market considered significant increases in the period from the beginning to the end of arbitrage operations as increasing risks. But they did not consider any changes in the contract duration within 15-45 days as increasing risks. Hence all contracts with duration in this interval can be analyzed uniformly as an analog of some averaged contract with one-month duration (30 days).

Remark: *the fact that there are no 1-5-day operations in the regression can be explained by the physical impossibility to perform such operations, since a new registration and transmission of stocks in the RTS system requires four trading days and an operator, after the purchase of a stock share, technically cannot deliver the share due to the futures contract.*

Statistical analysis of relations between the futures and the spot market
(The futures price of stocks as an estimate of the future spot price of stocks)

In an analysis of the futures market of any assets, we arrive at the following natural question: to what extent can the futures prices, at least on the average, estimate the

future price of an asset? In the present paper we restrict our study to the prices of the NK LUKOIL and RAO EES stocks that were most liquid in the Russian market from November 1996 to May 1998.

For a variable characterizing the proximity between the futures price and the future spot price of stocks, we take the difference of rates of return

$$RD(n)_t = r_a(n)_t - r_{st}(n)_t, \qquad (3.1)$$

where

$$r_a(n)_t = \frac{F_{t+n}(t) - P(t)}{P(t)} * \frac{365}{n}, \qquad r_{st}(n)_t = \frac{P(t+n) - P(t)}{P(t)} * \frac{365}{n}. \qquad (3.2)$$

In (3.1), (3.2) the number *n* is the duration of a contract in days, *P(t)* is the spot price on the day *t* without taking account of the spread between the purchase and sell prices, and $F_{t+n}(t)$ is the futures price of the contract with payment period of *n* days on the day *t*. Thus $RD(n)_t$ is the difference between the norm of income of the arbirage operation and the rate of return of the purchase and sell operation with stocks with time interval of *n* days. In other words, $RD(n)_t$ is the difference between the expected (from the viewpoint of the futures market) and the realized rate of return of the purchase and sell operation with stocks.

We assume that $RD(n)_t$ is a realization at time *t* of the random variable $RD(n) = r_a(n) - r_{st}(n)$. Then it is natural to expect that if, on the average, the future prices is correctly guessed by the futures market, then the mathematical expectation $E\{RD(n)\}$ must be equal to zero.

A statistical verification of this hypothesis is based on calculating the sampling average for each *n*:

$$\overline{RD(n)} = \frac{1}{N(n)} * \sum_t RD(n)_t = \overline{r_a(n)} - \overline{r_{st}(n)}.$$

The statistical hypothesis $E\{RD(n)\} = 0$ for each *n* was verified for NK LUKOIL and RAD EES stocks separately. The value of *n* varied from 1 to 90. The number of observed cases *N(n)* was different for different *n* and varied from 6 to 15 for NK LUKOIL and from 3 to 10 for RAO EES. The total number of cases $N_g = \sum_{n=1}^{90} N(n)$ was also different: N_g=515 for EESR and N_g=1050 for LKON.

There exist two different methods for verifying the original hypothesis. They depend on how the construction of the structure $RD(n)_t$ can be interpreted. The variable $RD(n)_t$ can be interpreted as a realization of the difference between two random variables $r_a(n)$ and $r_{st}(n)$. Then, to verify the original hypothesis, one must use the Student criterion for comparing the mean values of two normal samplings with different unknown variances. (It follows from the data available that the variables $r_a(n)$ and $r_{st}(n)$

do not have equal variances.) On the other hand, $RD(n)_t$ can be considered as a realization of the random variable *RD(n)* with unknown variance. In this case, by using another t-statistics, one can also verify the hypothesis that the mathematical expectation is zero.

We introduce the notation

$$s_a^2(n) = \frac{1}{N(n)-1} * \sum_t (r_a(n)_t - \overline{r_a(n)})^2 ,$$

$$s_{st}^2(n) = \frac{1}{N(n)-1} * \sum_t (r_{st}(n)_t - \overline{r_{st}(n)})^2 ,$$

$$\tilde{t}(n) = \frac{(\overline{r_a(n)} - \overline{r_{st}}(n))}{\sqrt{s_a^2(n) + s_{st}^2(n)}} * \sqrt{N(n)} .$$

It is a rather difficult problem to find the exact distribution function of the $\tilde{t}(n)$-statistics by comparing the mean values for two samplings. However, it is known that this distribution is close to the Student distribution with $[[\nu]]$ degrees of freedom, where $[[\nu]]$ is the integral part of the number ν, and

$$\nu = \frac{(s_a^2(n) + s_{st}^2(n))^2}{s_a^2(n) + s_{st}^2(n)} * (N(n)-1) .$$

It is also known that if $RD(n)_t$ is interpreted as a realization of the random variable *RD(n)* with its unknown variance, then the variable

$$t(n) = \frac{\overline{RD(n)}}{\sqrt{s_{RD}^2(n)}} * \sqrt{N(n)} , \text{ where } s_{RD}^2(n) = \frac{1}{N(n)-1} * \sum_t (RD(n)_t - \overline{RD(n)})^2 ,$$

satisfies the Student distribution with (*N(n)-1*) degrees of freedom.

Although the above formulas are different, the use of two Student criteria on the 90% confidence interval leads to the same conclusions.

Conclusions. For the LKOH stocks for all $n \neq 2;7;38;80$ considered, one cannot reject the hypothesis that the mean value is zero: $E\{RD(n)\} = 0$, while for the contract durations n=2;7;38;80, the hypothesis is rejected in favor of the hypothesis that $E\{RD(n)\} > 0$.

For the EESR stocks for all $n \neq 7;8;14$ considered, one cannot reject the hypothesis that the mean value is zero $E\{RD(n)\} = 0$, while for the contract durations n=7;8 the hypothesis is rejected in favor of $E\{RD(n)\} > 0$ and for n=14 in favor of $E\{RD(n)\} < 0$.

Thus, for the majority of durations of futures contracts, we cannot say that the futures market did not adequately «guess» relative changes in the stock price. Note that this conclusion does not contradict the fact that there are many possibilities for realizing profitable arbitrage operations.

4. Conclusions

1. The dynamics of prices in the Russian stock market during the period from September 1995 to September 1998 was mostly determined by the political events in Russia rather than by the state of affairs in the real sector of economics. The fact that the price of Russian stocks increased five times during the period from September 1995 to October 1997 and then returned to the original level in August 1998 is in no way related to the basic macroeconomics characteristics in Russia.

The only financial index that behaved in a similar way was the value of gold and currency reserves of the Central Bank of Russia. However, its variation was significantly less, it varies only by 1.4 times.

2. A relatively high correlation between the RTS index and the gold and currency reserves, as well as between the prices of 90- and 180-day GKO, makes it possible to assume that there exists a common stochastic trend between these pairs of time series. There is a probable explanation of this fact from the viewpoint of players, i.e., participants in the financial market. But attempts to find cointegration of these time series within the framework of the classical scheme meet the following principal difficulty: the time series of the first differences are nonstationary. However, their nonstationarity is not related to the «single root problem» in autoregression models.

3. At the same time, dynamics like to the Russian one was exhibited by the indices of the financial market in Venezuela. It is of interest to note that gold and currency reserves in Russia and in Venezuela in 1996-1998 were of similar dynamics and approximately of the same size. This fact also counts in favor of the hypothesis that there is a weak relation between the dynamics of the stock market and the dynamics of the real sector of economics in several developing countries.

4. Many theories describing the functioning of developed financial markets are based on the essential assumption that arbitrage operations are impossible. It is assumed that the market is organized so that the same arbitrage possibilities cannot exist for a long time because of the information circulating in the market and a large number of independent participants. But the situation in the Russian futures and spot markets was absolutely different from November 1996 to May 1998. There were rather long time periods during which pure arbitrage operations with borrowed capital, future contracts for stocks, and purchase-and-sell operations with stocks in the spot market could «bring money from air». There were even more possibilities for quasi-arbitrage operations in the futures and spot markets. The rate of return of these operations was significantly larger than the rate of return in the GKO market. (The difference between quasi-arbitrage and arbitrage is that the player has an initial financial capital either as some money resources or as a portfolio of stocks. In fact, this is an analog of bonds with a fixed interest but realized in two markets simultaneously.)

Although the number of such «profitable» days was large, the futures price of stocks could be used as an averaged estimate of the future price of stocks. More precisely, on the 90% confidence interval, the observed data do not allow one to say that there are distinctions in the mean between the supposed and actual relative changes in the stock prices almost for all contract durations form 1 to 90 days.

5. Apparently, participants of the financial market understand very well that the risks in any important operations in the Russian market are large. A comparatively short history of the Russian futures market is still another illustration of this fact. But the possibility of a rapid increase in the capital attracted large financial resources, both national and foreign. Nevertheless, the influence of these short-term portfolio investments

on the real sector of economics was very weak. And the real sector could not be attractive, since the rate of return in this market is low and the risks that credits will not be returned are high.

6. One can formulate a hypothesis that, for countries with transient economics, very high rates of increase in the stock index, which are not accompanied by a stable economical growth, must be accompanied by a significant flow of short-term capital from abroad and by existence of long-term arbitrage possibilities. These phenomena possess a positive feedback.

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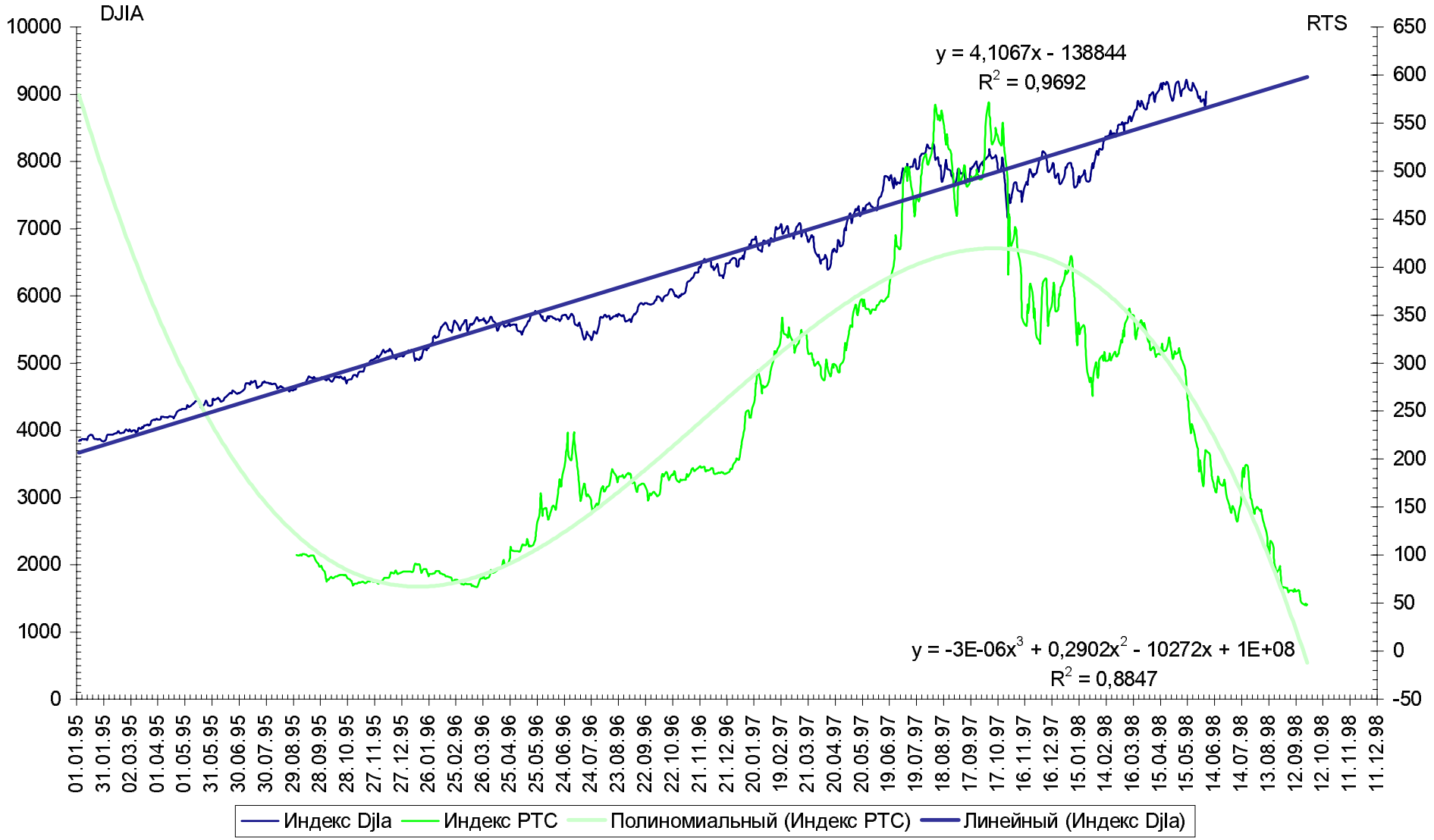
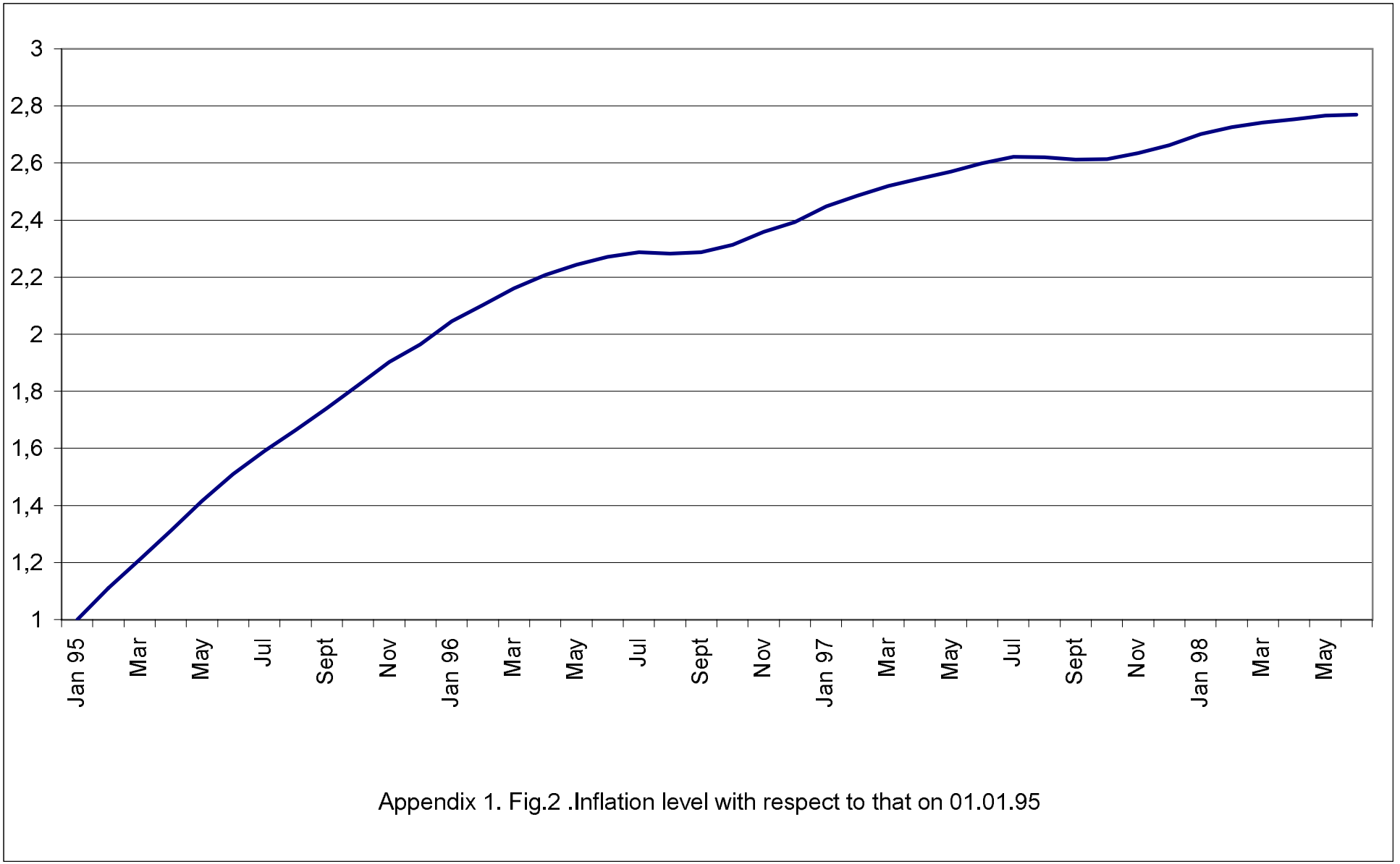
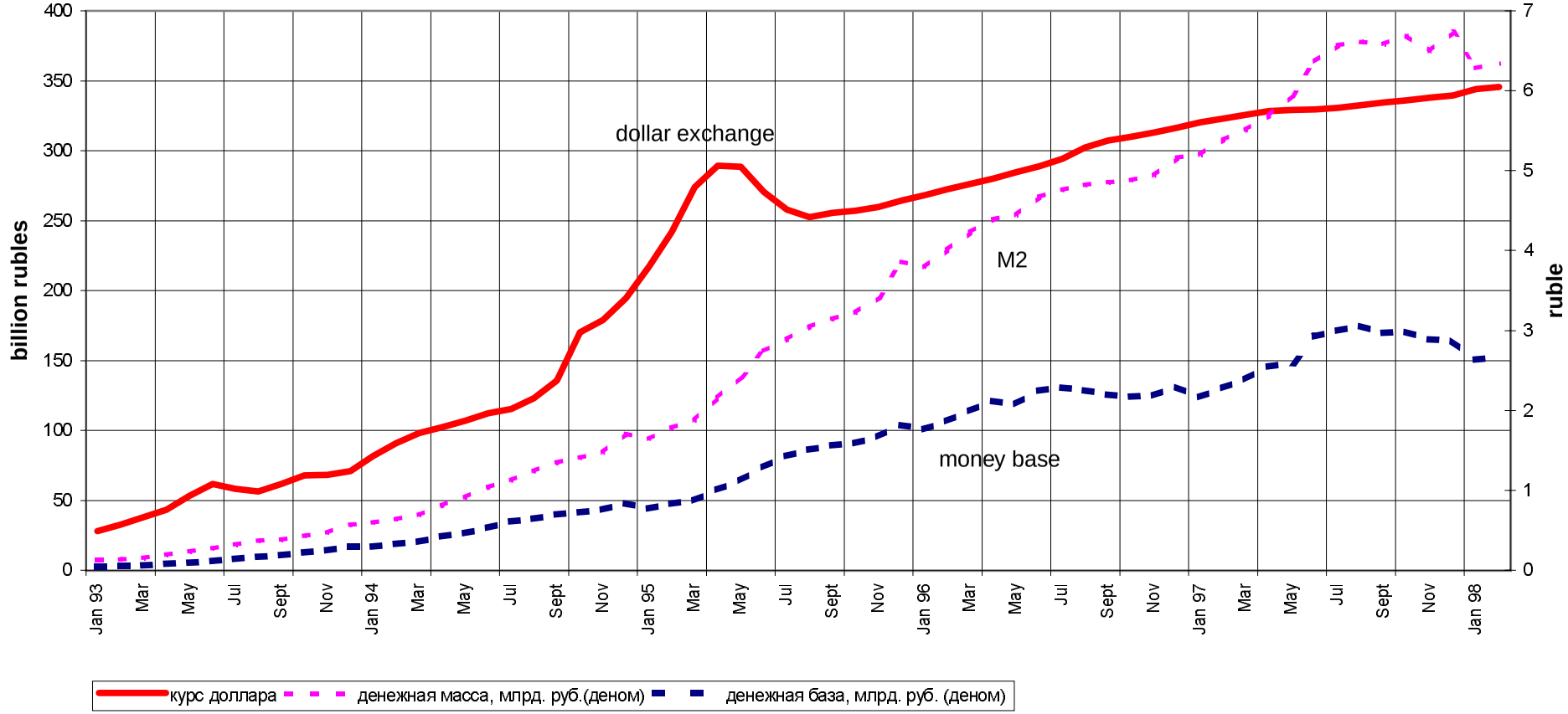


Appendix 1. Fig 1. Index RTS & DJIA



Appendix 1. Fig.2 .Inflation level with respect to that on 01.01.95



Appendix 1. Fig.3 Money agregate dynamics and dollar exchange rate dynamics

Appendix 2

In 1998 in Russia the situation in the market of stock exchange contracts of various terms was such that the most part of this market was concentrated in Moscow. The leader of the Moscow futures market in volumes of operations was the Russian Stock Exchange. From 1994 to May 1998 the following contracts were traded in this market:

1) currency futures;
2) three types of GKO contracts:
3) futures contract for the following stocks:
   - 3a) packet of 100 stocks of NK «Lukoil» (LKON - delivery data),
   - 3b) packet of 1000 stocks of Mosenergo (MSNG - delivery data),
   - 3c) packet of 1000 stocks of Rostelekom (RTKM - delivery data),
   - 3d) packet of 10000 stocks of RAO EES (EESR - delivery data),
   - 3e) packet of 10 stocks of Sberbank of RF (SBER - delivery data).

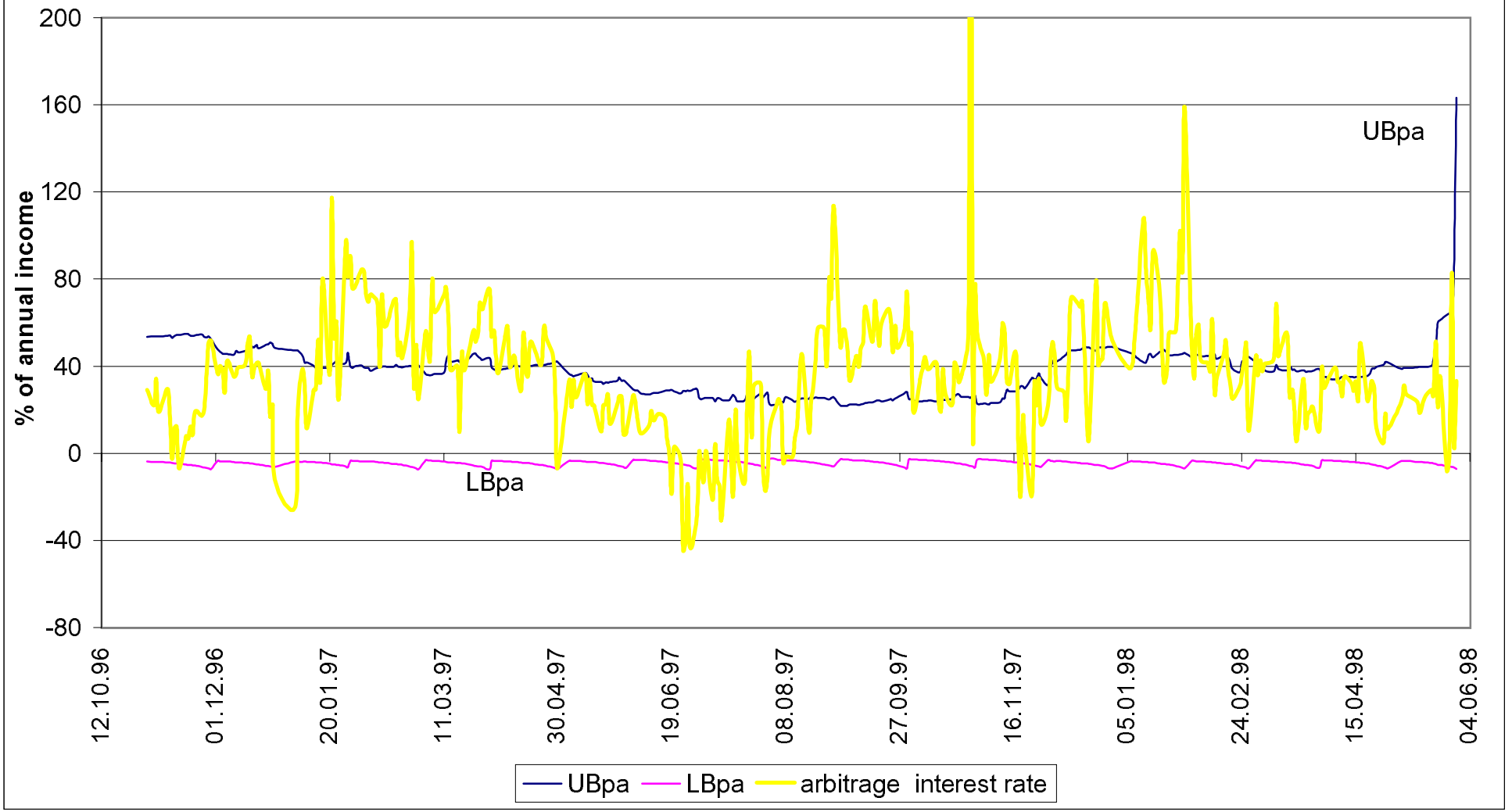
The contracts for stocks are executed with respect to the positions that remain open on the 14th of each month after the trade of these contracts is finished.

The annual trade volumes for all types of futures contracts are given in Table F1 on the basis of the data published in Russian Stock Exchange Vestnik, 1, 1998.

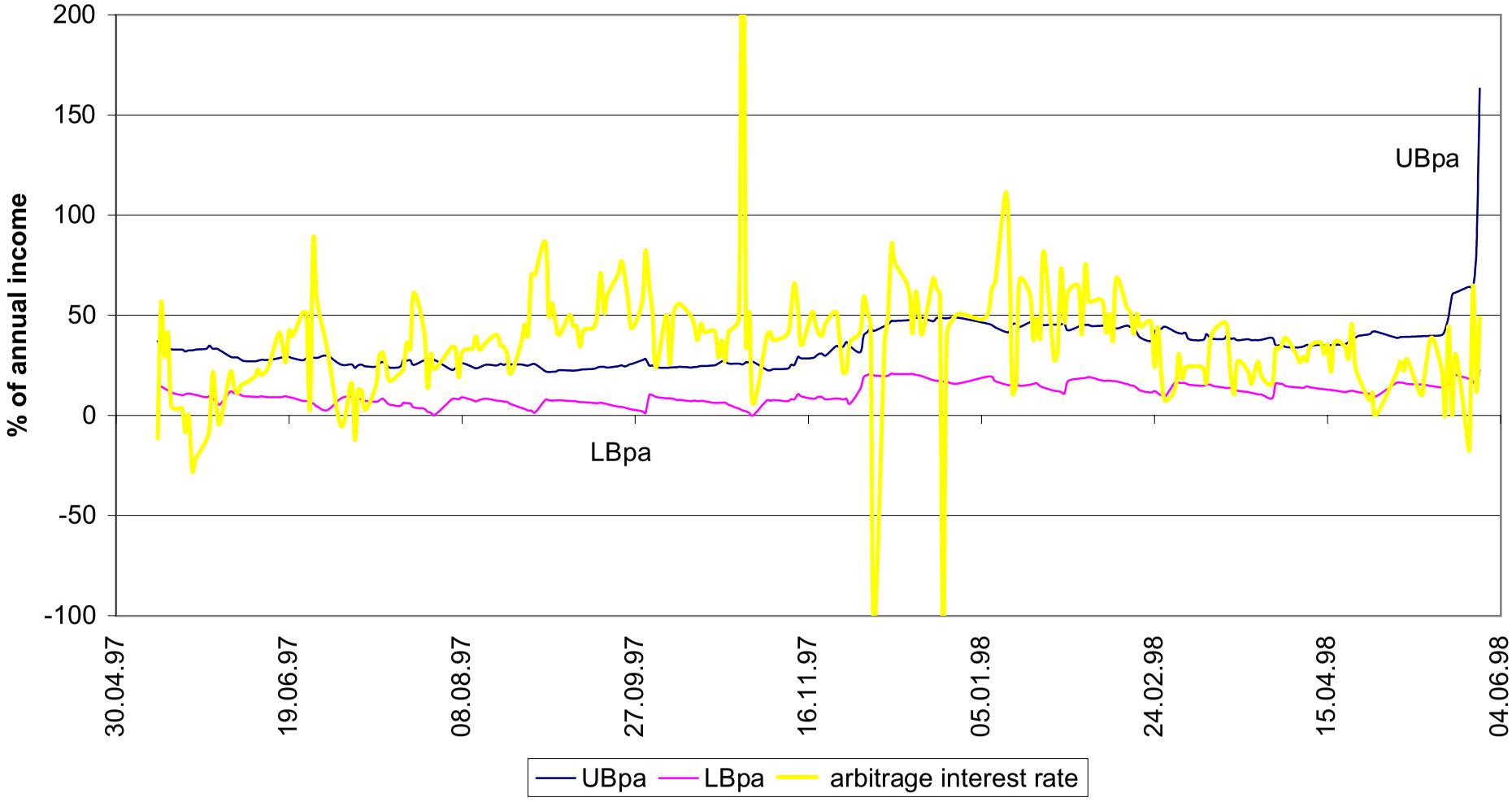
Table F1

Year	Annual trade volume						
	general	Type of contracts					
		Currency		GKO		Stocks	
	$million	$mill	%	$mill	%	$mill	%
1994	37	37	100	–	–	–	–
1995	2138	2129	99.5	9	0.5	–	–
1996	5335	862	16	4446	83.5	27	0.5
1997	33852	–	–	3340	10.5	28512	89.5

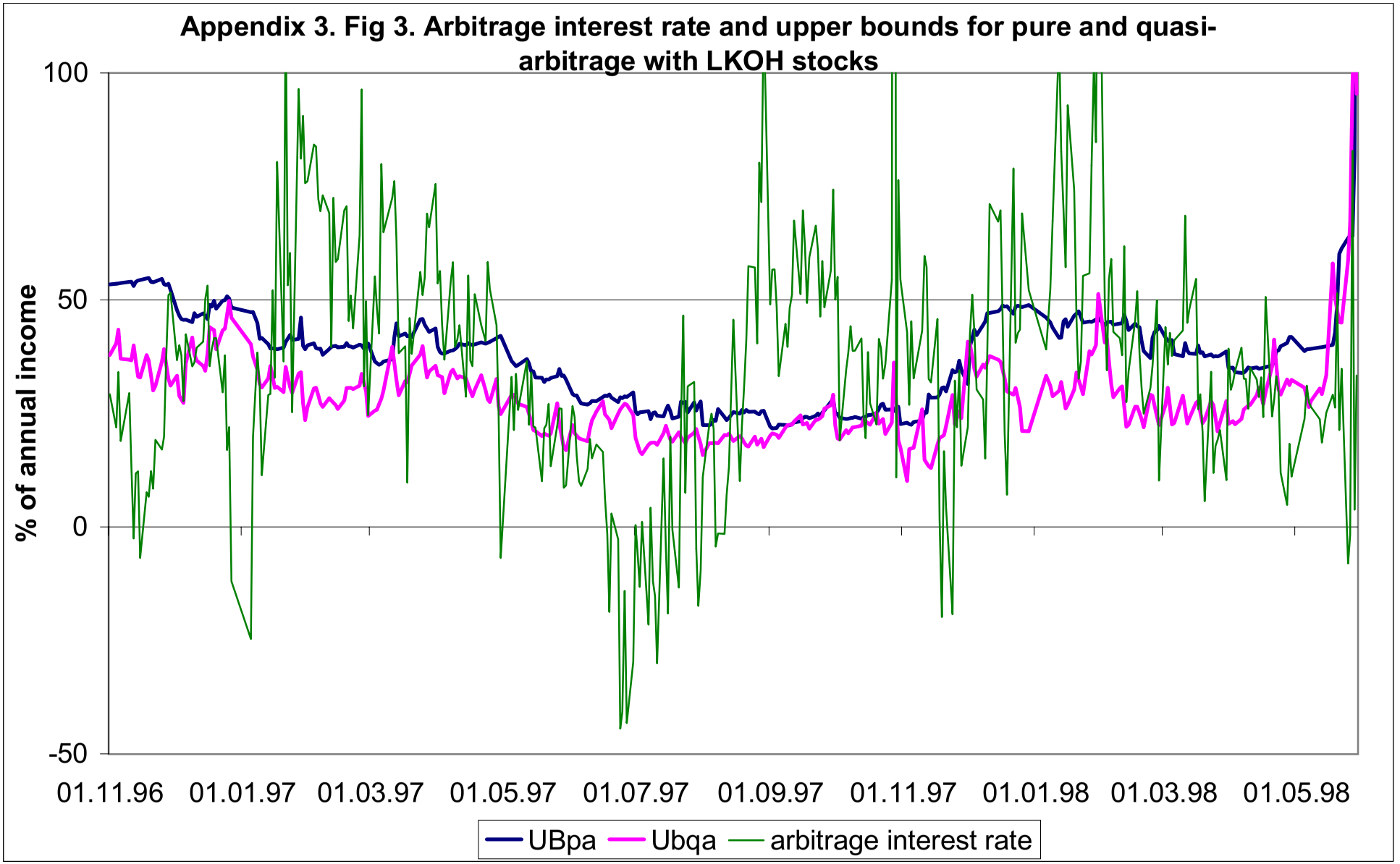
Appendix 3. Fig 1. Upper and Lower Bounds for pure arbitrage and arbitrage interest rate with LKOH stocks



Appendix 3. Fig 2. Upper and Lower Bounds for pure arbitrage and arbitrage interest rate with EESR stocks



Appendix 3. Fig 3. Arbitrage interest rate and upper bounds for pure and quasi-arbitrage with LKOH stocks



Appendix 3. Fig 4. Arbitrage interest rate and upper bounds for pure and quasi- arbitrage with EESR stocks